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TLS-Based Prefiltering Technique for Time-Domain ARMA Modeling

by

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Modeling time-series with linear pole-zero AutoRegressive-Moving Average (ARMA) models has numerous applications in signal processing. This problem is in general non-linear and most ARMA modeling techniques are iterative in nature. The Iterative Prefiltering (IP) method has the advantage of computing potential non-minimum phase representations which may be useful in time-domain modeling. The original IP minimization procedure is an ill-conditioned problem which has classically been solved using a least-squares approach. This work presents a modification of the classical IP technique in which the least-squares iteration step is replaced by a Total Least Squares (TLS) step to take advantage of the statistical properties of the TLS method. Results show that improvements in the modeling performances may be obtained by using the TLS-based IP method when modeling signals distorted by white Gaussian noise.

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TLS-BASED PREFILTERING TECHNIQUE FOR TIME-DOMAIN ARMA MODELING

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1. Introduction

Modeling time-series with linear pole-zero AutoRegressive-Moving Average (ARMA) models has numerous applications in signal processing. This problem is in general non linear and most ARMA modeling techniques are iterative in nature. The Iterative Prefiltering (IP) method has the advantage of computing potential non-minimum phase representations which may be useful in time-domain modeling. The original IP minimization procedure is an ill-conditioned problem which has classically been solved using a least-squares approach. This work presents a modification of the classical IP technique in which the least-squares iteration step is replaced by a Total Least Squares (TLS) step to take advantage of the statistical properties of the TLS method. Results show that improvements in the modeling performances may be obtained by using the TLS-based IP method when modeling signals distorted by white Gaussian noise.

Section 2 presents a review of the TLS method, and illustrates its main advantages as compared to the LS method. Next, Section 3 presents the classical Iterative Prefiltering method and the proposed TLS-based IP method. Finally, conclusions are given in Section 4.

2. Total Least Squares Problem

2.1 Introduction

The Total Least Squares method (TLS) has been introduced recently as a substitute to the classical Least Squares (LS) method for solving overdetermined linear systems of equations when all data involved in the computation are corrupted with noise (errors) [1,2]. Consider the problem of solving the overdetermined system of equations $A\underline{x}=\underline{b}$, where A has dimension m*n and $\underline{b}$ has dimension m*1. The LS solution $\underline{x}_{ls}$ is obtained by minimizing the Euclidean norm $\|A\underline{x}-\underline{b}\|_2$. Solving for the LS solution is equivalent to solving
the following minimization problem:

$$\mathit{minimize}_{\underline{b}' \epsilon \mathbf{R}^m} \quad \|\underline{b}-\underline{b}'\|_2$$

$$\mathit{subject\ to} \quad \underline{b}' \in R(A)$$

The above equation is satisfied when $\underline{b}$' is the orthogonal projection of $\underline{b}$ onto R(A). Thus, the LS vector $\underline{b}$' can be viewed as the perturbation of $\underline{b}$ so that $\underline{b}$' can be generated by the range (or the column space) of A. As a consequence, the LS solution assumes that errors can occur only in the vector $\underline{b}$ and not in the matrix A. However, this assumption is not always realistic as errors such as sampling, modeling, and instrumentation errors may force inaccuracies in A also. The TLS takes into account the fact that errors may occur both in the matrix A and in the vector $\underline{b}$ and solves the following minimization problem:

$$\mathit{minimize}_{[\hat{A};\hat{\underline{b}}]\ \epsilon\ \mathbf{R}^{m*(n+1)}} \quad \|[A;\underline{b}]-[\hat{A};\hat{\underline{b}}]\|_2$$

$$\mathit{subject\ to} \quad \hat{\underline{b}} \in R(\hat{A})$$

Thus, the TLS vector $\hat{\underline{b}}$ can be viewed as resulting from the perturbation both from the columns of

A and the vector $\underline{b}$ so that $\hat{\underline{b}}$ belongs to $R(\hat{A})$. An illustration of the differences between the LS and the TLS solution for a system with column space of dimension two is shown in Figure 1.

2.2 SVD-Based Solution of the TLS Problem

The Singular Value Decomposition (SVD) is used to solve the TLS problem [1,2]. Recall that the system to be solved is of the form $A\underline{x}=\underline{b}$, which can be reformulated as:

$$[A;\underline{b}]\begin{bmatrix}\underline{x}\\-1\end{bmatrix}\approx\underline{0}.$$

The TLS solution is found from the SVD of the augmented matrix

$$C = [A;\underline{b}] = U\Sigma V^{H}.$$

The solution is said to be generic or non generic depending on the numerical characteristics of the eigenvalue matrix Σ and the right singular vector matrix V.

2.2.a TLS Generic Solution

The TLS solution $\underline{x}_{TLS}$ is unique and said to be generic if the singular value $\sigma_n > \sigma_{n+1}$ and $v_{n+1,n+1} \neq 0$. In such a case the solution is given by:
$\underline{x}_{TLS} = -1/v_{n+1,n+1}[v_{1,n+1},\ldots,v_{n,n+1}]^t$.

When the p smallest singular values are equal, i.e., $\sigma_{n-p} > \sigma_{n-p+1} = \ldots = \sigma_{n+1}$, the TLS solution to the linear system is not unique, and any linear combination of the singular vectors associated with the multiple minimum singular value can be chosen provided that it is normalized properly. The resulting minimum norm solution can be shown to be equal to [1,3]

$$\underline{x}_{TLS} = -\frac{\hat{V}_2\underline{c}}{\underline{c}^t\underline{c}} = \frac{\hat{V}_1\underline{g}}{1-\underline{g}^t\underline{g}}.$$

Matrices $\hat{V}_1$ and $\hat{V}_2$ are submatrices of the matrices of the right singular vector matrices V_1 and V_2, where V_2 is the matrix of right singular vectors associated with the multiple minimum singular value, and V_1 contained all the other right singular vectors. $\hat{V}_1$ and $\hat{V}_2$ are defined as:

$$V_2 = \begin{bmatrix}\underline{c}^t\\ \hat{V}_2\end{bmatrix}, \ and \ V_1 = \begin{bmatrix}\underline{g}^t\\ \hat{V}_1\end{bmatrix},$$

where the vectors $\underline{c}^t$ and $\underline{g}^t$ respectively represent the first row of V_2 and V_1.

Again, the TLS solution exists only if all $v_{n+1,i} \neq 0$ for $i=n-p+1,\ldots,n+1$. Van Huffel showed that no generic solution exists only when the matrix A is nearly rank deficient, or when the set of equations is conflicting [4]. The set of equations is said to be conflicting when $\sigma_{n-p+1} \approx \ldots \approx \sigma_{n+1}$ are large. In such a case, trying to model the data using a linear model is inaccurate, and a better

option should be chosen by the user. When $\sigma_{n-p+1} \approx \ldots \approx \sigma_{n+1}$ are small, two options are possible. The first option for the user is to remove dependent columns of A to reset the problem so that the resulting modified A matrix is regular, and to solve a generic TLS problem. How to pick such columns is addressed in [2,14]. The second option is to solve a nongeneric TLS problem which is obtained from adding additional constrains on the generic TLS problem in order for it to be solvable.

2.2.b The Nongeneric TLS Problem

The nongeneric TLS problem is addressed in details in Van Huffel et al. [4,5,7]. The resulting solution is given by:

$$\underline{x}_{TLS} = -1/v_{n+1,n-p}[v_{1,n-p},\ldots,v_{n,n-p}]^t,$$

when $\sigma_{n-p} > \sigma_{n-p+1} = \ldots. \sigma_{n+1}$, $v_{n+1,i}=0$ for $i = n-p+1,\ldots, n+1$, and $v_{n+1,n-p} \neq 0$.

2.3 Applications of the TLS to Signal Processing Problems

Numerous researchers in Signal Processing have reformulated problems in terms of the TLS technique. Applications can be found in array processing [11], in system modeling [10], and in frequency estimation of sinusoidal signals [9,10-13] for example. Results show that improvements in the performances can be obtained when reformulating LS problems in a TLS set-up for the applications mentioned above. This result motivated our investigation of the TLS technique to time-series data modeling. ARMA modeling of time-series is a non-linear problem which has been extensively studied in the literature [17]. The Iterative Prefiltering (IP) method is an iterative linearized formulation which was originally proposed by Steiglitz and McBride to identify linear system transfer functions [15,16], and the IP method can be reformulated to model time-series data distorted with noise. Note that the method is not insured to converge to the optimal solution and De Moor showed that it converges to a suboptimal solution only [18]. Nevertheless, simulation results have shown that useful models can still be obtained using the IP method [19].

3. The Iterative Prefiltering Method

3.1 Introduction

The Iterative Prefiltering method attempts to model a time-series data with an ARMA(P,Q) system using a time-domain approach by minimizing the error between the data and the impulse response of the system to be estimated. Using the Z-domain, the model function is given by:

$$H(z) = \frac{B(z)}{A(z)},$$

where

$$A(z) = \sum_{k=0}^{P} a(k)z^{-k} \quad and \quad B(z) = \sum_{k=0}^{P} b(k)z^{-k}.$$

The error function for the problem is defined by

$$E(z) = X(z) - \frac{B(z)}{A(z)} = \frac{X(z)A(z)-B(z)}{A(z)}. \tag{1}$$

Non-linearity in the problem is due to the denominator A(z) in the error function E(z). The problem may be linearized by replacing the error function given in eq. (1) with the iterative error function

$$E^{(i)}(z) = \frac{X(z)A^{(i)}(z)-B^{(i)}(z)}{A^{(i-1)}(z)},$$

where indices (i) and (i-1) refer to iterations (i) and (i-1). At each iteration the quantities $A^{(i)}(z)$ and $B^{(i)}(z)$ are chosen to minimize the error $E^{(i)}(z)$. The estimation of the ARMA transfer function is done in the time domain using the time series x(n), for n=0,...,N-1, by rewriting $E^{(i)}(z)$ in the time-domain as

$$E^{(i)}(n) = \underline{x}^{(i)H}(n) * \underline{a}^{(i)} - \underline{h}^{(i)H}(n).\underline{b}^{(i)}, \qquad n=0,...,N-1 \tag{2}$$

with:

$$E^{(i)}(n) = \underline{x}^{(i)}(n)^{H}.\underline{a}^{(i)} - \underline{h}^{(i)H}(n).\underline{b}^{(i)} \quad n=0,...,N-1$$

$$\underline{x}^{(i)}(n) = [x^{(i)}(n),...x^{(i)}(n-P)]^{T}$$
$$\underline{h}^{(i)}(n) = [h^{(i)}(n),...,h^{(i)}(n-Q)]^{T}$$
$$\underline{a}^{(i)} = [1,a^{(i)}(1),...,a^{(i)}(P)]$$
$$\underline{b}^{(i)} = [b^{(i)}(0),...,b^{(i)}(Q)]$$

where $h^{(i)}(n)$ is the impulse response of $H^{(i)}(z)=1/A^{(i)}(z)$, and $x^{(i)}(n)$ represents the output of x(n) through the filter $H^{(i)}(z)$. Eq. (2) expressed in matrix form becomes:

$$\underline{E}^{(i)} = \begin{vmatrix} 0 & ... & ... & x^{(i)}(0) \\ 0 & ... & x^{(i)}(0) & x^{(i)}(1) \\ \vdots & ... & \vdots & ... \\ x^{(i)}(N-P) & ... & x^{(i)}(N-2) & x^{(i)}(N-1) \end{vmatrix} \begin{vmatrix} a^{(i)}(P) \\ a^{(i)}(P-1) \\ \vdots \\ 1 \end{vmatrix} - \begin{vmatrix} h^{(i)}(0) & 0 & ... & 0 \\ h^{(i)}(1) & h^{(i)}(0) & ... & 0 \\ \vdots & ... & ... & \vdots \\ h^{(i)}(N-1) & ... & ... & h^{(i)}(N-Q) \end{vmatrix} \begin{vmatrix} b^{(i)}(0) \\ b^{(i)}(1) \\ \vdots \\ b^{(i)}(Q) \end{vmatrix} \tag{3}$$

which may be rewritten as:

$$\underline{E}^{(i)} = X^{(i)}\underline{a}^{(i)} - H^{(i)}\underline{b}^{(i)}. \tag{4}$$

Equation (4) may be decomposed into two parts using the orthonormal projection matrix P onto the column space of $X^{(i)}$, as noted by McClellan and Lee [17], which leads to:

$$\underline{E}^{(i)} = P^{\perp}X^{(i)}\underline{a}^{(i)} + (PX^{(i)}\underline{a}^{(i)} - H^{(i)}\underline{b}^{(i)})$$
$$with \quad P = H^{(i)}[H^{(i)H}H^{(i)}]^{-1}H^{(i)H} \tag{5}$$

Recall that the IP iteration minimizes the expression $\|\underline{E}^{(i)}\|_2$, which leads to the following minimization problem [17]:

- (a) minimize $P^{\perp}X^{(i)}\underline{a}^{(i)}$ subject to $a^{(i)}(0)=1$,
- (b) solve $\underline{b}^{(i)}=[H^{(i)H}H^{(i)}]^{-1}H^{(i)H}X^{(i)}\underline{a}^{(i)}$

Step (a) above may be rewritten as:

$$[C,-\underline{d}][a^{(i)}(P),\ldots,1]^T=\underline{0},$$

where

$$C=P^{\perp}X^{(i)}[1{:}N,1{:}P],\ \underline{d}=P^{\perp}X^{(i)}[1{:}N,P+1].$$

Step (a) of the IP method has classically solved using the classical LS approach, which accounts for errors in $\underline{d}$ only. However, errors occur both in C and $\underline{d}$. A better fit of the data can be obtained by taking into account for errors in both in C and $\underline{d}$, as the TLS set-up allows us to do. Thus, the TLS-Based IP iteration solves for (a) using a TLS approach [20], which leads to the following minimization problem:

- (a) Minimize $P^{\perp}X^{(i)}\underline{a}^{(i)}$ subject to $a^{(i)}(0)=1$, using a TLS method.
- (b) solve $\underline{b}^{(i)}=[H^{(i)H}H^{(i)}]^{-1}H^{(i)H}X^{(i)}\underline{a}^{(i)}$.

3.2 Data Scaling

3.2.a Introduction

Van Huffel has shown that when errors in the TLS matrix and right-hand-side vector are uncorrelated with zero-mean and equal variance, then under mild conditions the TLS solution is a strongly consistent estimate of the true solution of the unperturbed system [4]. For the problem considered, errors in the data result from errors in the signal x(n) to be modeled. Assuming the data to be modeled is distorted with Gaussian white noise, the errors in $H^{(i)}$ are correlated as they are obtained as the output of an AR linear system. However, consistency in the TLS estimate may still be retained by using the Generalized TLS (GTLS), as proposed originally by Van Huffel [2,4,17]. Thus, data scaling is needed to insure diagonal error covariance matrices, and the problem becomes to estimate the error covariance matrices $F^{(i)}$ and $G^{(i)}$ at each iteration given by:

$$F^{(i)}=E[N^{H(i)}N^{(i)}],$$
$$G^{(i)}=E[N^{(i)}N^{(i)H}],$$

where $N^{(i)}$ represents the errors (noise contribution) in $[C,-\underline{d}]$ at iteration (i).

3.2.b Computation of the Matrix $F^{(i)}$

Assume that the noisy signal x(n) is defined by x(n)=s(n)+w(n), where w(n) is wss white noise, and s(n) is an ARMA signal. The noise correlation matrix $F^{(i)}$ may be expressed in terms of the correlation matrix $R_W^{(i)}$ obtained from the AR process $1/A^{(i)}(z)$:

$$\begin{aligned} F^{(i)} &= E[N^{(i)H}N^{(i)}] = E[(P^{\perp}W^{(i)})^{H}(P^{\perp}W^{(i)})] \\ &= E[W^{(i)H}P^{\perp H}P^{\perp}W^{(i)}] \\ &= E[W^{(i)H}P^{\perp}W^{(i)}] = E[W^{(i)H}(I-P)W^{(i)}]. \\ &= E[W^{(i)H}W^{(i)}]-E[W^{(i)H}W(i)] \\ &= \quad R_W^{(i)} \quad - \quad E_W^{(i)} \end{aligned} \tag{6}$$

Computations show that the components of $E_W^{(i)}$ in equation (6) can be expressed as:

$$E_W^{(i)}(l,k)=\sum_{q,j=1}^{n} P_{q,j}r_W^{(i)}(|l+q-j-k|), \tag{7}$$

where $P_{i,j}$ is the $(i,j)^{th}$ component of the projection matrix P defined earlier, and $r_W^{(i)}(n)$ is the correlation function obtained when passing the white noise w(n) through the AR system with transfer function $1/A^{(i)}(z)$. Using the fact that the noise distortion w(n) is assumed to be white, $r_W^{(i)}(|l+q-j-k|)$ is non zero only when $l+q-j-k=0$. Thus, the double summation in equation (7) reduces to a single summation. Furthermore, the correlation function $r_W^{(i)}(k)$ can easily be computed using the results by Dugre et. al. [6] who proposed an algorithm to generate a covariance sequence from its AR coefficients.

3.2.c Computation of the matrix $G^{(i)}$

Again the error covariance matrix $G^{(i)}$ may be expressed in terms of the correlation matrix $R_W^{(i)}$ obtained from the AR process $1/A^{(i)}(z)$ obtained at iteration (i) by:

$$\begin{aligned} F^{(i)} &= E[N^{(i)}N^{(i)H}] = P^{\perp}E[W^{(i)}W^{(i)H})]P^{\perp} \\ &= P^{\perp}R_W^{(i)}P^{\perp} \end{aligned} \tag{8}$$

3.2.d GTLS Solution

Applying the results presented by Van Huffel, the general TLS-based IP iterative solution for $\underline{a}^{(i)}$ at iteration (i) is then given by solving:

$$\left(R_G^{(i)+}[C,-\underline{d}](R_F^{(i)})^{-1}\right)\left(R_F^{(i)}\begin{vmatrix} a^{(i)}(P) \\ \vdots \\ a^{(i)}(1) \\ 1 \end{vmatrix}\right) \approx \underline{0} \quad with \quad R_F^{(i)}=chol(F^{(i)}), \quad R_G^{(i)}=U\Sigma^{+}U^{H},$$

with:

$$\Sigma^{+}=diag[\sqrt{\sigma_1},\ldots,\sqrt{\sigma_r},0,\ldots 0]$$

where r is the rank of $R_G^{(i)}$. Next, the estimate for $\underline{b}^{(i)}$ can be obtained by replacing $\underline{a}^{(i)}$ by its value in the expression:

$$\underline{b}^{(i)}=[H^{(i)H}H^{(i)}]^{-1}H^{(i)H}X^{(i)}\underline{a}^{(i)}$$

3.3 Simulation Results

The TLS-based IP method is implemented on time-series data generated by an ARMA(3,4) in additive white noise and its performances compared with that of the LS-based IP method. In both cases, initial estimates for the iterative procedures are obtained using Yule Walker equations for the poles and Shank's technique when solving for the zeros. The resulting noisy signal is modelled with an ARMA(5,6) using a sequence of length 60. The noise variance is chosen equal to 2 and the ARMA parameters are chosen equal to:

$\underline{a}_p$=[1,-2,1.6725,-0.4613]
$\underline{b}_p$=[1,-.9,0.04,0.3960,-0.2912]

Figure 2.a shows the noise-free signal, the noisy signal, and the estimated signal obtained during the first 5 iterations using the classic IP method. Power spectral estimates obtained for the noise-free signal, the noisy signal, and that obtained with the initial ARMA coefficients estimated using Yule Walker equations and Shank's method, are shown in Figure 2.b. Figures 3.a and 3.b represent the same estimates obtained when using the proposed TLS-based IP scheme. Note that Figures 2 and 3 represent the best performances obtained during the first 5 iterations for both schemes. Figure 4 and 5 represent the true pole (a_p) and zero (b_p) locations, and estimated poles (a_{ip}) and zeros (b_{ip}) locations obtained when using the IP and TLS-based IP schemes. This example illustrates the fact that the bias in the estimated pole locations obtained using the TLS-based technique is usually smaller than that obtained using the LS-based IP method when the SNR is medium to high. However, we noted that when the SNR is low, no distinct improvement is noted consistently when using the proposed technique.

4. Conclusions

This study has investigated the application of the TLS problem in the Iterative Prefiltering method for time-domain ARMA modeling. Results show that the pole bias is usually smaller when using the TLS-based scheme than when using the classical LS-based IP method, when SNR are medium to high. We note that, similarly to the LS-based method, the TLS-based IP method is not

TLS-based IP method. The main drawback in the proposed TLS-based IP method is the computational load increase.

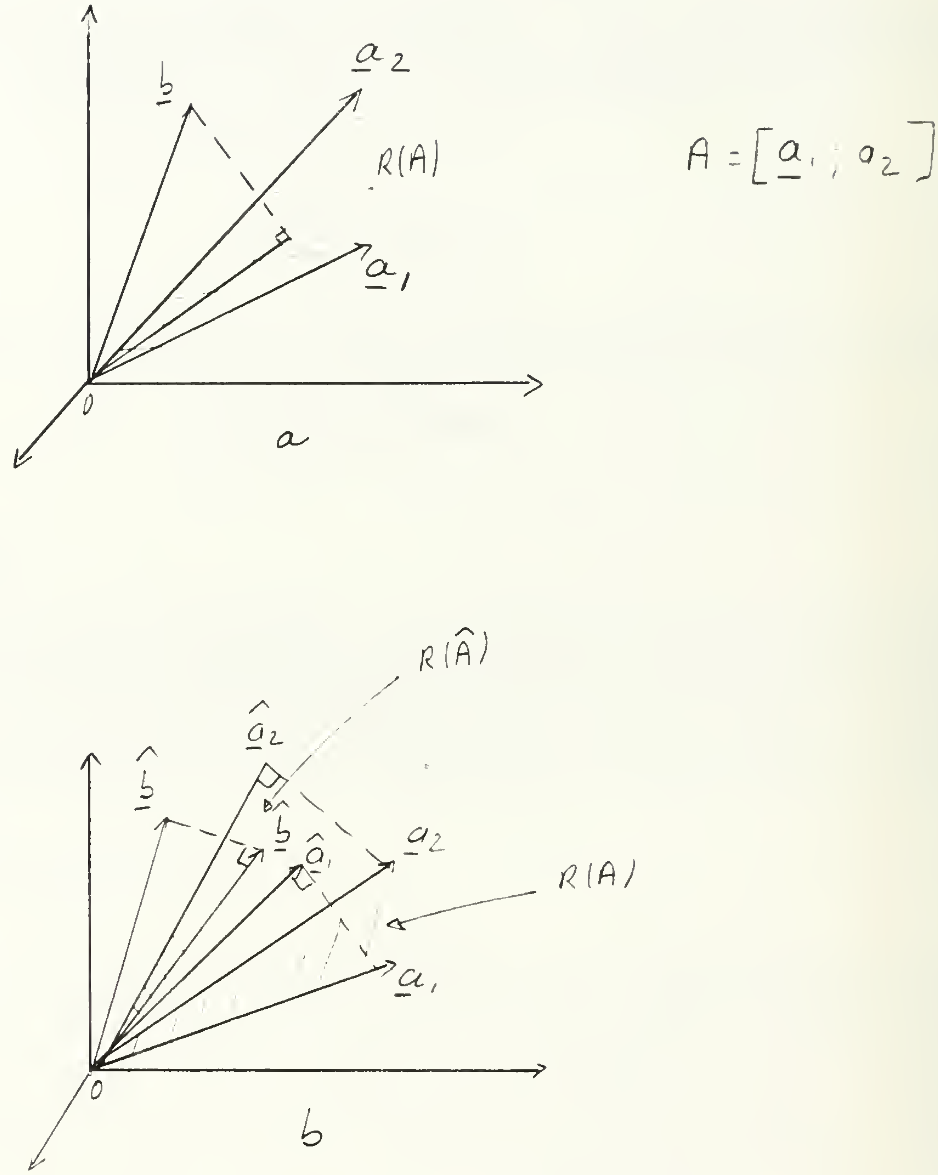
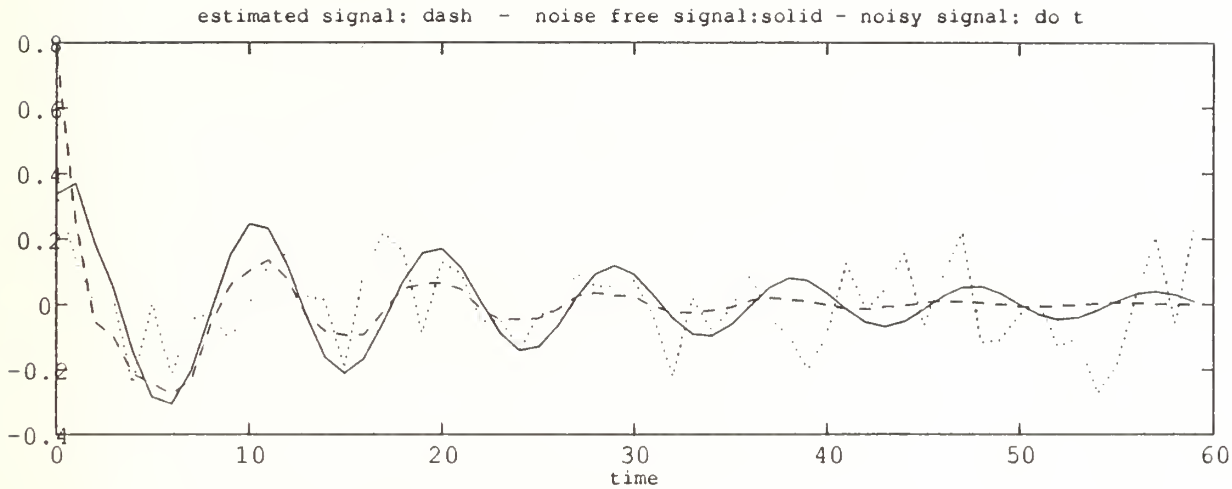


Figure 1.a) LS solution to $A\underline{x}=\underline{b}$; b) TLS solution to $A\underline{x}=\underline{b}$.



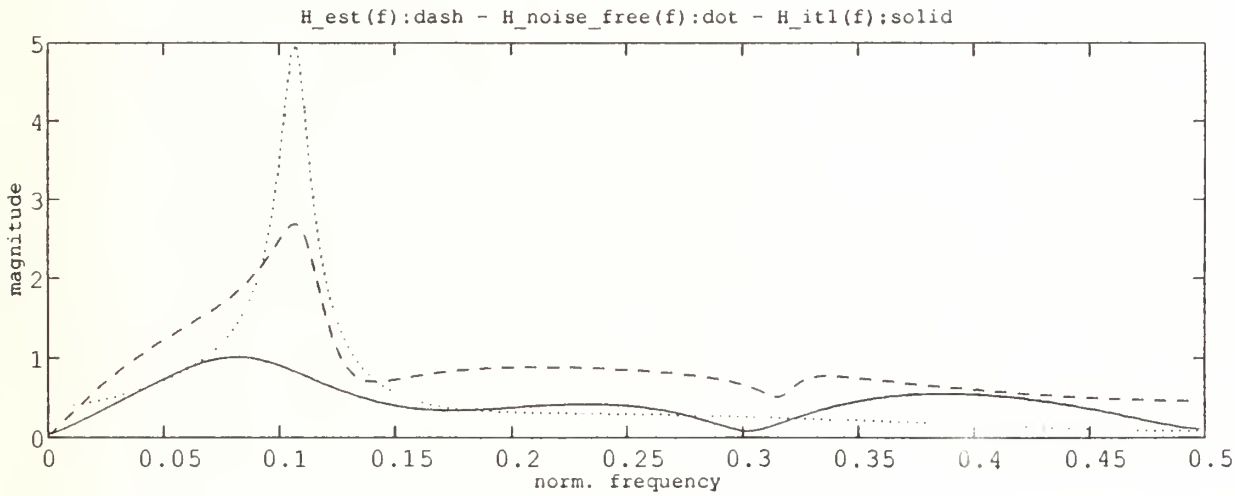


Figure 2.a) Noise-free signal, noisy signal, LS-based IP estimated signal; b)Spectra: H_noise_free: noise-free signal, H_est:LS-based IP signal, H_it1: initial estimate.

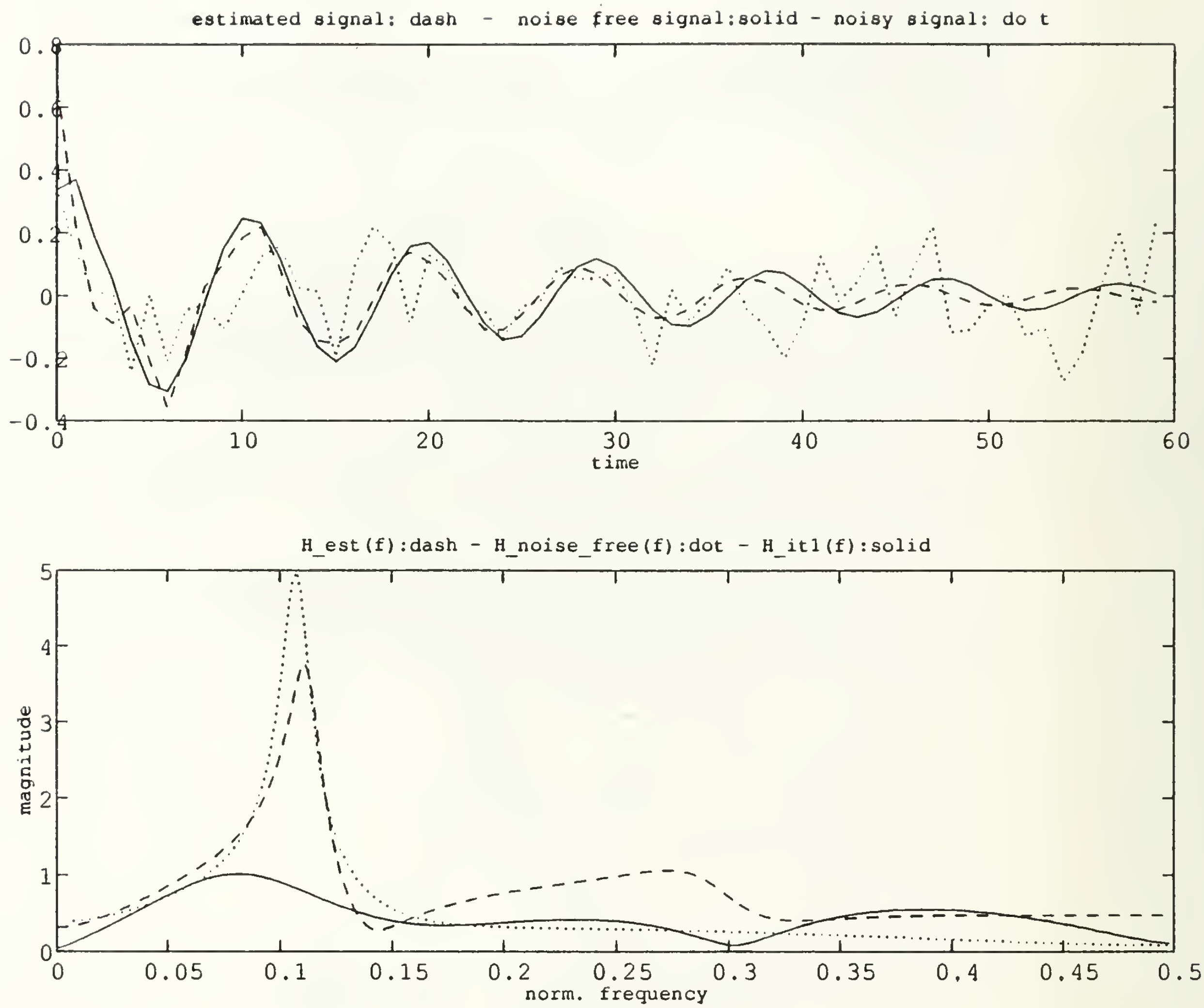


Figure 3.a) Noise-free signal, noisy signal, TLS-based IP estimated signal; b)Spectra: H_noise_free: noise-free signal, H_est: TLS-based IP signal, H_it1: initial estimate.

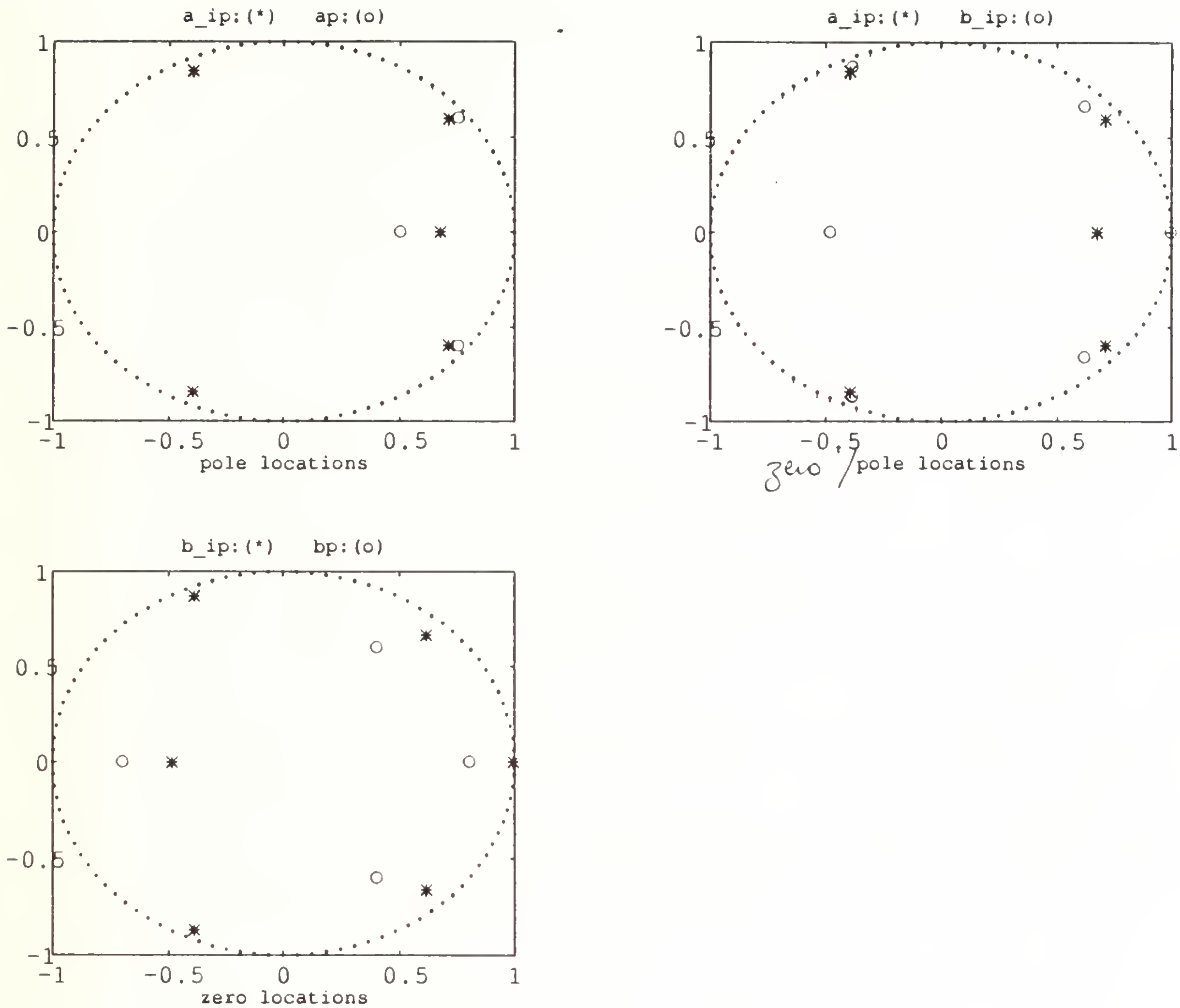


Figure 4. True pole (a_p) and zero (b_p) locations; LS-based IP estimated pole (a_{ip}) and zero (b_{ip}) locations.

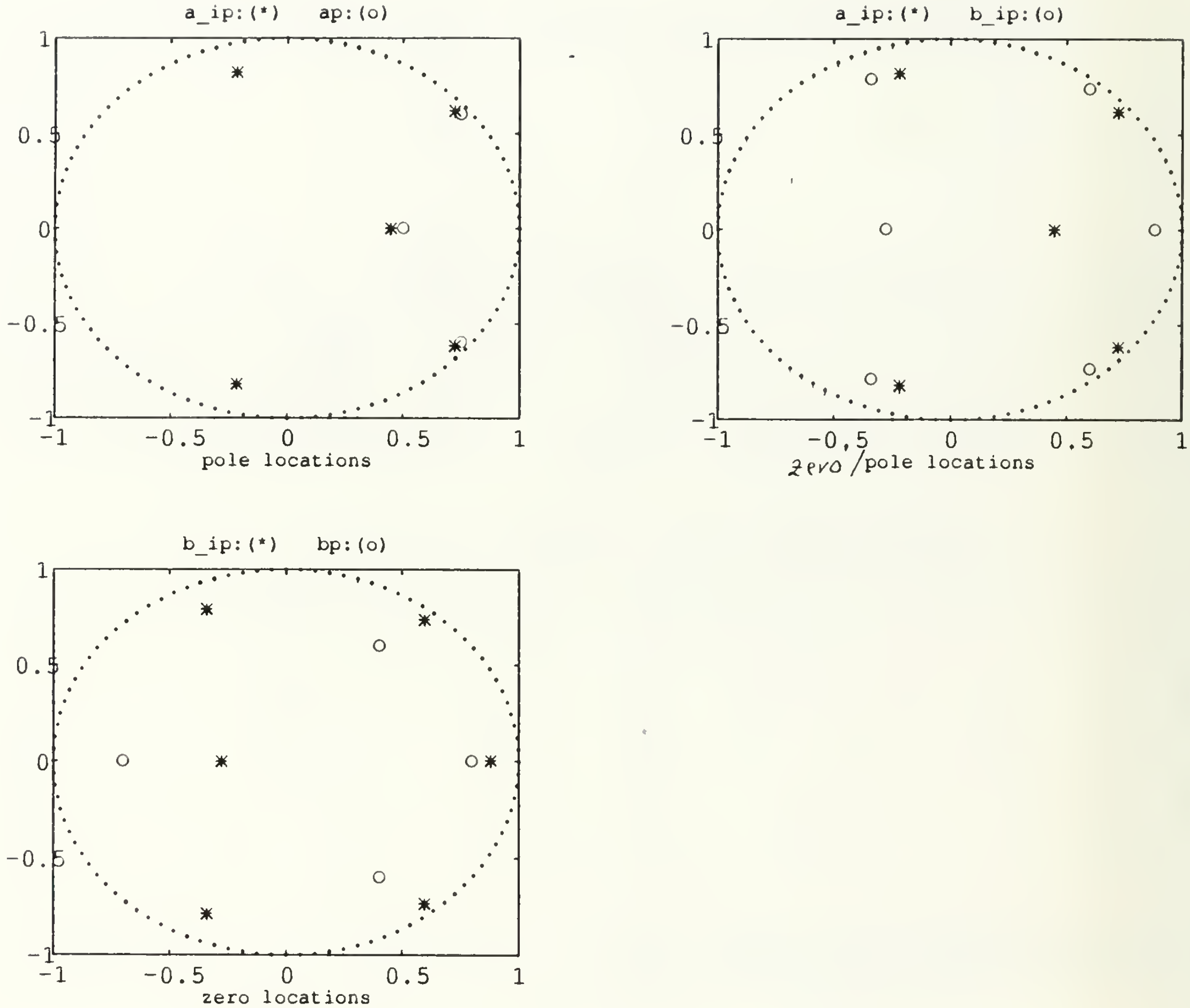


Figure 5.True pole (a_p) and zero (b_p) locations; TLS-based IP estimated pole (a_{ip}) and zero (b_{ip}) locations.

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